Merge Sort

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History of Merge Sort

- Merge sort was one of the first sorting algorithms proposed for computing, brought forward by John von Neumann in 1945.
- Merge sort becomes one of the first divide-and-conquer sorting algorithms
- The algorithm has roots in card-sorting machines of the late 19th century

Von Neumann

The Merge Sort Algorithm

4	3	6	2	7	1	8	5

```
procedure: mergesort ($a_1$, $a_2$, ..., $a_n$: array of integers)
if (length of array ≤ 1), then
          return array
left := left half of input array
right := right half of input array
left := mergesort(left)
right := mergesort(right)

final := empty array
while (left is not empty and right is not empty) then
          if (first element of left ≤ first element of right), then
                    append first element of left to final
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          else if (first element of right < first element of left), then
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if (left is still not empty), then
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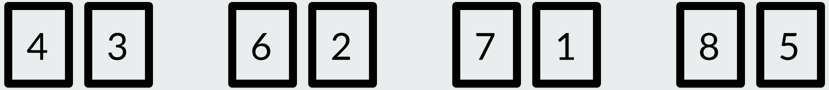
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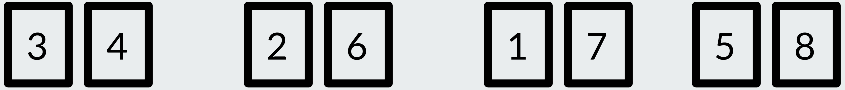

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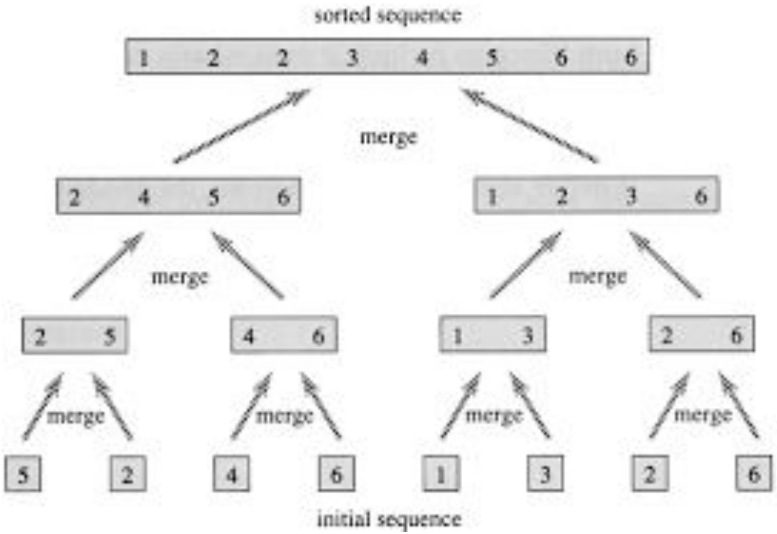
Complexity of Merge sort

Algorithm	Merge Sort	Insertion Sort (a simple sorting algorithm)
Best-case time complexity	$\Omega(nlogn)$	$O(n^2)$
Worst-case time complexity	$O(nlogn)$	$O(n)$ comparisons, $O(1)$ swaps
Space complexity	$O(n)$	$O(n)$

Alternative Merge Sort: Iterative Version

- We just examined a **top-down** implementation of Merge Sort, which uses recursion to sort the entire list
- However, we can implement a **bottom-up** Merge Sort, **without using recursion**
- To do this, we immediately start merging and sorting instead of first dividing the sequence into groups



Bottom-up Merge Sort

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procedure: mergesort ($a_1$, $a_2$, ..., $a_n$: array of integers)
list := input array
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for (k=0, k<log2(length of list), k++), do
        for (j=0, j<n, j+=2^(k+1)), do
                merge(list, final, j, 2^k)
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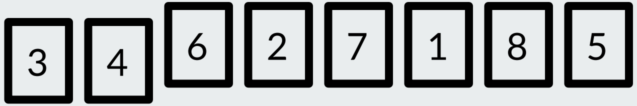

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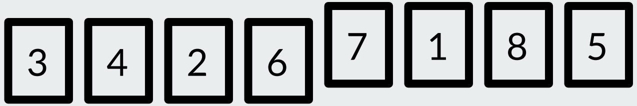

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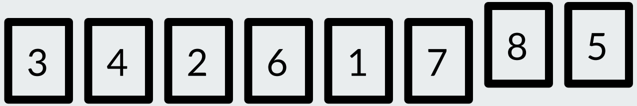

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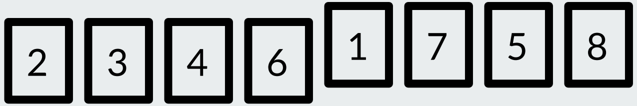

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